Before doing any math, ask: **What are the units?** And what does the quantity $A$ measure? Those two questions can prevent a surprising number of errors when reading a graph, building a model, or comparing two reports.

The notation $A(t)$ says that a quantity named $A$ depends on time $t$. But it does not tell us, by itself, when the clock starts or what is being counted. A(t) might be the number of packages in a warehouse, the dollars in an account, or the amount of water in a tank. If one person defines $t=0$ at midnight and another defines $t=0$ at the start of a delivery shift, they can use the same formula and still be talking about different moments.

The recording period start matters because it establishes the reference point for time. To interpret $A(t)$ correctly, define the recorded quantity, identify the start time, and keep the units attached. Then ask whether the question is about a total or a rate. Those are different questions.

## First define what $A$ records

Imagine a shipping dashboard that reports $A(t)$, the number of packages processed by time $t$. Before calculating anything, rewrite the situation in one clean sentence:

**"$A(t)$ is the cumulative number of packages processed since the recording period began, $t$ hours after that start."**

That sentence gives the function a job. It says what is being recorded, that the quantity is cumulative, and how $t$ is measured. If the recording period begins at 6:00 a.m., then $t=0$ means 6:00 a.m.; $t=3$ means 9:00 a.m. The value $A(3)$ is the total processed during the period from 6:00 to 9:00 a.m., assuming the cumulative count was reset to zero at the beginning.

Now change the meaning slightly. Suppose $B(t)$ records the total number of packages processed so far that day, with the clock starting at midnight. At 9:00 a.m., $B(9)$ may include packages processed before the shift began. Even though $A$ and $B$ both count packages, their values answer different questions because their recording periods start at different times.

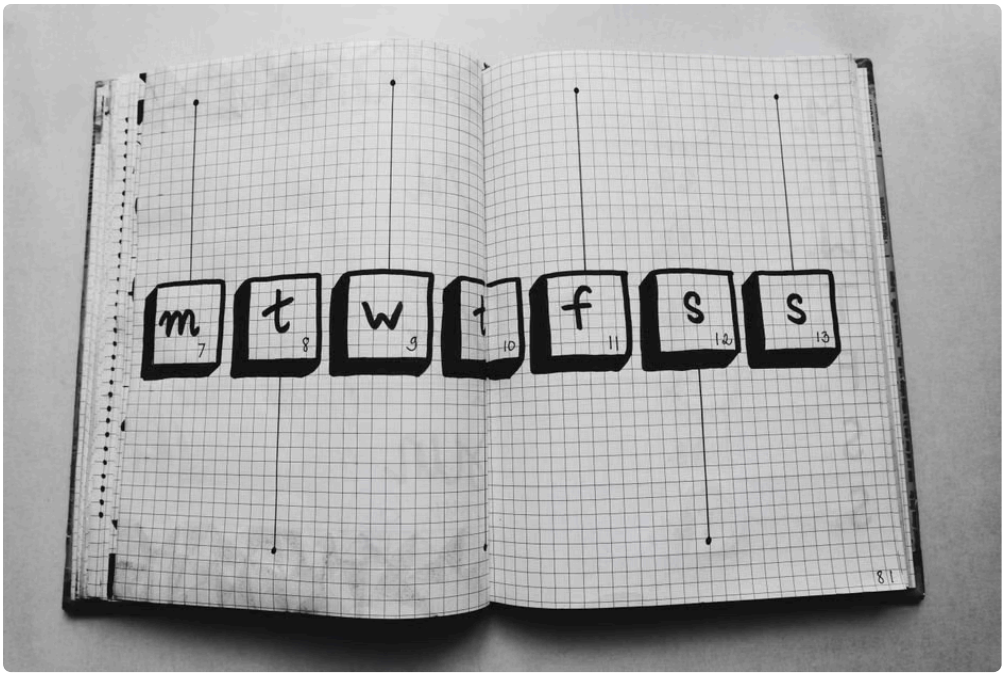
A useful definition should answer these questions:

- **What is the quantity?** Packages, dollars, liters, or something else?
- **Is it a total or a rate?** For example, packages processed so far versus packages processed per hour.
- **When does the recording period start?** A calendar date, a shift start, a delivery arrival, or another event?
- **What are the units of $t$?** Seconds, minutes, hours, or days?

## Defining $t=0$ is choosing a reference

Setting $t=0$ is not just a notational convenience. It chooses the reference time from which every other time is measured. This is similar to choosing where a ruler begins: the distance between two marks does not change, but the coordinate labels do.

Suppose a delivery arrives at 8:00 a.m., and the warehouse begins tracking its inventory at that moment. We can define $t=0$ https://activecalculus.org/author/Running-Totals-Rates-of-Change as 8:00 a.m. Then $t=2$ hours refers to 10:00 a.m. If another report uses midnight as its reference, that same moment is $t=10$ hours. The clock values differ, but the real-world instant is identical.




In general, if the original time variable is \(t\) and we move the reference point forward by \(c\) time units, a new variable might be defined by \(s=t-c\). The same event then receives a different coordinate. The function also needs to be described using that new reference. For instance, if \(A(t)\) gives the inventory at clock time \(t\), then in the shifted time coordinate we can write \(A_\textnew(s)=A_\textold(s+c)\). The physical inventory has not changed; the time labels have.

This is why we say the **function depends on its reference**. A formula without its reference point may be incomplete for practical interpretation. It may tell us how values relate to time while leaving it unclear which real-world time each input represents.

### A quick timeline check

Before evaluating \(A(4)\), translate the input into a clock time. If \(t=0\) is 6:30 a.m. and \(t\) is measured in hours, then \(t=4\) means 10:30 a.m. If \(t\) is measured in minutes, \(t=4\) means 6:34 a.m. Never mix hours and minutes without saying so.

This simple translation is especially helpful when a dashboard combines systems. A shipping report may record elapsed minutes after a truck arrives, while an inventory report uses the clock time of day. The two timelines need a clear conversion before their values can be compared.

## Totals and rates answer different questions

A total tells us how much has accumulated. A rate tells us how quickly a quantity changes. These are related, but they are not interchangeable.

If \(A(t)\) is a cumulative package count, its units are packages. The average rate of change over an interval is

\[ \fracA(t_2)-A(t_1)t_2-t_1. \]

If \(t\) is measured in hours, the units of this quotient are packages per hour. The numerator is a change in the total number of packages; the denominator is elapsed time. Calling \(A(t)\) itself “the rate” would be incorrect: it is a total, measured in packages, not packages per hour.

For example, suppose a shift begins at 6:00 a.m., and the cumulative totals are \(A(0)=0\) packages and \(A(3)=ninety\) packages. From 6:00 to 9:00 a.m., the average processing rate is

\[ \fracA(3)-A(0)3-0 =\frac90\text packages3\text hours =30\text packages per hour. \]

The total at 9:00 a.m. is 90 packages. The average rate over the three-hour interval is 30 packages per hour. Those are two distinct answers, with different units.

The start time matters here because it helps determine which accumulation is included. If the count began at midnight rather than at 6:00 a.m., then the total at 9:00 a.m. might include three extra hours of activity. The rate between two appropriately chosen times might still be calculated, but the total’s meaning would differ.

## Average rate depends on the interval

An average rate of change always belongs to an interval. It is not enough to say, “The average rate was 30 packages per hour,” without saying over what period that average was calculated. The morning average from 6:00 to 9:00 may differ from the average from 9:00 to noon.

Consider this table. The times are hours after the recording period begins at 6:00 a.m.

Elapsed time, \(t\) (hours) Clock time Cumulative packages, \(A(t)\) 0 6:00 a.m. 0 2 8:00 a.m. 50 5 11:00 a.m. 140

From 6:00 to 8:00 a.m., the average rate is \(50/2=25\) packages per hour. From 8:00 to 11:00 a.m., it is \((140-50)/(5-2)=30\) packages per hour. From 6:00 to 11:00 a.m., it is \(140/5=28\) packages per hour. Each calculation is valid, but each answers a different interval question.

This distinction matters in work settings. A manager might ask, “How many packages were processed by 11:00?” That is a total question. Another manager might ask, “How quickly did processing happen between 8:00 and 11:00?” That is an average-rate question. Write down the interval before calculating, and include units in the result.



## What if the quantity changes in jumps?

Many real-world totals do not change smoothly. Inventory may remain unchanged until a shipment arrives, then jump upward. A package count may stay flat between scans, then increase when a batch is recorded. In those cases, a step graph is often more honest than a smooth curve.

Suppose a warehouse has 120 items at 9:00 a.m., receives a shipment of 40 items at 10:00 a.m., and has 160 items afterward. A step graph shows the quantity staying at 120 until the receipt is recorded, then jumping to 160. A straight-line connection from 120 to 160 would imply that the inventory increased gradually between 9:00 and 10:00. That may describe a useful estimate in some models, but it does not describe what the recorded data says if the entire shipment arrived at once.

Mark the axes with units and state the recording convention. For example:

- The horizontal axis is time in hours after the recording period began.
- The vertical axis is inventory in items.
- The value at \(t=0\) is the inventory at the start of the period.
- A jump marks a recorded event, such as a delivery or a batch scan.

Then interpret average rates carefully. Across an interval containing a jump, the average rate of change may be meaningful as a summary: total change divided by elapsed time. But it does not imply that items arrived at a steady pace throughout the interval.

## Units are a safeguard, not decoration

Units help us catch errors and explain results. If \(A(t)\) is measured in dollars and \(t\) in months, then an average rate of change is measured in dollars per month. If \(A(t)\) is a balance, however, we still need to say whether it is an account balance at a moment or the amount deposited during a period. Both might use dollars, yet they describe different quantities.

A unit check can reveal a mistaken operation. Subtracting two cumulative package totals gives a change measured in packages. Dividing by elapsed hours gives packages per hour. Adding a total measured in packages to a rate measured in packages per hour is not meaningful without a model that explains how the rate is used over time.

For finance dashboards, clarify whether a plotted value is a balance, a transaction amount, or a rate of spending. A balance of \$500 is not the same as spending \$500 per week. Also specify the time reference: Is the balance recorded at the start of the day, after the day’s transactions, or at the close of business?

## Use MathJax and Runestone to make the ideas visible

MathJax can render expressions such as \(\fracA(t_2)-A(t_1)t_2-t_1\) clearly in a web page, making it easier for readers to see which values are totals and which calculation produces a rate. Good math rendering helps, but it cannot supply a missing definition. The words around the expression must still identify \(A\), its units, the time units, and the recording period start.

Runestone, an interactive publishing platform used for learning materials, can support short activities where readers change the time reference and observe how input values are relabeled. An activity might ask learners to identify the real clock time corresponding to \(t=3\), or compare average rates on two intervals. Interactivity is most useful when the prompt makes the assumptions visible rather than hiding them.

## A practical checklist for reading \(A(t)\)

When you encounter a time-dependent quantity in a chart, report, or textbook, work through this sequence:

1. **Rewrite the situation in one clean sentence.** State what \(A(t)\) measures and whether it is a total, balance, or other quantity.

2. **Find the recording period start.** Identify the event or clock time that defines $t=0$.
3. **Check the units.** Record the units for both $A$ and $t$, and convert minutes and hours when necessary.
4. **Translate the input.** Turn a value such as $t=4$ into a real clock time using the stated reference.
5. **Name the interval.** For an average rate, identify both endpoints and calculate change in the total divided by elapsed time.
6. **Respect the data's shape.** Use a step graph when recorded events create jumps; do not draw smooth connections that claim changes happened between timestamps unless that is an intentional model.

The start time matters because $A(t)$ is more than a formula: it is a measurement tied to a reference. Once the quantity, recording period start, and units are clear, you can tell whether a question asks for a total at a moment or an average rate over an interval. That clarity makes the mathematics easier—and makes the answer far harder to misread.